Value and Price of Production: A Note on Winternitz' Solution[1]

That the "transformation problem" remains controversial even in its purely formal aspects is testimony to the isolation of two streams in the history of economic doctrine—the "orthodox" economics of respectability and the "bootleg" political economy of Marx. This isolation has resulted in two jargons with the most confusing differences of meaning at points of apparent similarity and in the refusal, by and large, of partisans of either doctrine to consider seriously any method previously contaminated by adherents of the other. For example, orthodox economists have practically ignored technological change while Marxists have generally shied away from the mathematical method.

The note by Winternitz is a step in the right direction. His equations show that if value and price of production are defined according to Marx, there exists a simple transformation connecting the two. The transformation is independent of any equilibrium conditions (which appear redundantly in Bortkiewicz) and preserves the total of values. The "transformation problem" in the formal sense of linking value and price of production is seen to be practically trivial mathematically. Winternitz thus clears up an artificial confusion initiated by Bortkiewicz, whose pseudo-mathematical mystifications bore little or no relation to the basic problem posed by Marx.[2] The real problems in this field are not

[1] J. Winternitz, "Values and Prices," Economic Journal, 1948, pp. 276 *et seq.*

[2] When redundant algebraic verbiage is cleared away, the solution of Bortkiewicz (which was reproduced by Sweezy) is seen to be the same as that of Winternitz except that Winternitz defines price of production so that the total of values equals the total of prices of production, whereas Bortkiewicz substitutes an *ad hoc* assumption. It is Winternitz, of course, who is consistent with Marx.

to be found in the alleged difficulty of relating value and price of production, but in relating these two concepts to actual prices, to the actual exchanges, which, it must be remembered, are not involved in the definitions of value and price of production in terms of labour time involved in the productive process.[1] Nevertheless, clarity on formal matters (practically matters of definition) is a prerequisite to the use of a theoretical structure in dealing with significant problems.

We mention, therefore, a few points which appear to remain unclear and which illustrate the terminological and methodological difficulties indicated in the first paragraph. In the first place, there is in the note by Winternitz an apparent identification of price of production with price in the ordinary sense of the ratio at which an actual exchange takes place. But the two are different concepts, just as distinct as value and price. In orthodox usage value is simply another term for price, or, more strictly speaking, price is value per unit commodity. With Marx, " value " is a social relation dependent only on the technology and sociology of production. " Value " and price not only may not coincide quantitatively, but their coincidence would imply conditions which " could not exist without abolishing the entire system of capitalist production." [2] To emphasise the distinction, Marx pointed out that a commodity may have a price without having a value.[3] But the " price of production " is with Marx a form of value, derivable from value, and like it equally distinct from price.[4] Not only may price deviate from price of production, but a commodity may have a price without having a price of production. Marx maintained that value and price coincide under conditions of simple commodity production and under certain average conditions of competitive capitalism. He assumed that value and price coincide throughout Vols. I and II of *Capital*, an assumption equivalent to considering only average and aggregate

[1] Marx deals with this question in *Capital*, Vol. III, Ch. X. He considers in detail the relations between market-value (average value of the commodities of a certain type) and market-price (average sale price of the same) and concludes with the remark that the same discussion holds with suitable modifications when price of production is substituted for market-value.

[2] *Capital*, Vol. III, p. 181.

[3] *Capital*, Vol. I, p. 115.

[4] " In volumes I and II we were dealing only with the *values* of commodities. Now we have dissected this value on the one hand into a *cost-price*, and on the other we have developed out of it another form, that of *price of production* of commodities." (Vol. III, p. 192. Marx' italics.) The entire discussion in Ch. X, Vol. III, makes clear that Marx is considering three distinct entities: value, price of production and price. For example : " It is a matter of definite differences in quantity between market-prices and market-values, and, further, prices of production " (p. 229).

relations. In Vol. III Marx argues that price of production and price coincide, or more precisely that market-price of production serves as a norm around which market-price oscillates.[1] In writing about socialism, some writers have argued that prices should be set according to values, while others have maintained that they should be set according to prices of production. It is these statements about relationships between the value concepts (value, market-value, price of production, etc.) and actual prices (price, market-price, etc.) which pose the significant economic problem. The relationship between value and price of production is hardly more than a formal preliminary.[2] The solution of Winternitz is limited to the relation of value and price of production, but the use of " price " in place of " price of production " suggests otherwise.

With respect to Winternitz' mathematical derivation, it should be noted that his method is independent not only of the conditions of simple reproduction (as he correctly points out) but also of the context of the division of the economy into three branches. The derivation may be interpreted as an example, for the case $n = 3$, of the way in which values and prices of production of the n commodities in the economy are related. This interpretation would, in addition, be closer to Marx, who used calculations primarily as illustrations to accompany verbal arguments which combined process and cross-section analysis in a way which could hardly be fitted to the mathematical techniques available even to-day.[3] Winternitz' derivation is limited strictly to the relation between values and prices of production at an instant, actual prices and economic processes in time not being directly involved.

Since Winternitz clarifies one mathematical confusion, it is unfortunate that he repeats another. Bortkiewicz started his discussion of the transformation problem with given values of constant capital (capital goods), variable capital (labour-power),

[1] " We call it price of production. It is, as a matter of fact, the same thing which Adam Smith calls *natural price*, Ricardo *price of production*, or *cost of production*, and the physiocrats *prix nécessaire*, . . ." (Vol. III, p. 233.)

[2] " In these capitalist processes it is not a question of the formal conversion of the value of commodities into prices, not a question of a mere change of form " (*ibid.*, p. 229). " The real difficulty lies in the question : How is this equalisation of profits into an average rate of profit brought about, seeing that it is evidently a result, not a point of departure " (*ibid.*, p. 205).

[3] Marx was acquainted with the calculus, but like Marshall and many other economists, he used more mathematics in working out his results than he did in presenting them. See, in this connection, Engels' prefaces to *Capital*, Vols. II and III.

surplus value and value of output in each of three divisions of production: those producing capital goods, wage goods and luxury goods. He then derived from these the prices of production of the same items. He also found a formula for the average rate or profit under exchange at prices of production. This formula involved the original values only from the first two divisions of production, and Bortkiewicz drew the conclusion that the profit rate is, in fact, independent of the division producing luxury goods. But Bortkiewicz assumed that the original values satisfied the equilibrium conditions of "simple reproduction," from which it follows that the values in the first two divisions are related to those in the third, and hence the rate of profit is not independent of division three. One could, by utilising his equilibrium conditions, rewrite the formula for the profit rate so as to involve the values from division three.[1] Although Winternitz' solution is in other respects different from that of Borkiewicz, he derives the same formula for the average rate of profit. Since Winternitz does not assume that the values satisfy equilibrium conditions, one cannot say as above that the profit rate does depend on all three divisions. On the other hand, one cannot conclude that it is independent of division three. It all depends on whether the original values in the three divisions are, in fact, independent or not. They may still be related even if equilibrium conditions are not satisfied, and they are not made independent merely by our failure to write down some explicit relations between them. If the independence of the values is explicitly assumed or, which I think unlikely, proved by some economic argument, then the independence of the profit follows from the formula. As it stands, the conclusion is merely the result of an implicit assumption that variables are independent until proved otherwise.

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